Electromagnetic-thermal dosimetry of the human brain - Application to transcranial magnetic stimulation (TMS)

D. Poljak

University of Split, FESB

The scientific man does not aim at an immediate result. He does not expect that his advanced ideas will be readily taken up. His work is like that of a planner for the future. His duty is to lay foundation of those who are to come and point the way.

NIKOLA TESLA
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- WM topic and description:
- EM and thermal dosimetry: Previous work
- EM and thermal dosimetry: Ongoing work
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- Concluding remarks and future work
This WM deals with a EM–thermal dosimetry model of the human brain.

The EM model is based on the *surface integral equation (SIE)* formulation - the equivalence theorem + appropriate boundary conditions for the case of lossy dielectric object of an arbitrary shape.

The thermal dosimetry model of the brain - Pennes’ equation for heat transfer in biological tissue.
WM topic and description

• The goal – accurate SAR distribution assessment and related temperature increase in tissues.

• A particular feature of the proposed EM-thermal dosimetry: the rigorous model of transcranial magnetic stimulation (TMS) based on the SIE approach.

• To the best of our knowledge similar approach in modeling TMS has not been previously reported, albeit integral equation methods are seeing a revival in CEM community.
To account for the inductive and capacitive effects, as well as the propagation effects, often neglected within quasi-static approximation, a model of a lossy homogeneous brain has been derived (the equivalence theorem + appropriate boundary conditions for the electric field).

The model aims to provide an accurate representation of the TMS induced fields and currents, respectively.

The numerical solution of the SIE is carried out by an efficient scheme of the method of moments (MoM).
EM and thermal dosimetry: Previous work

Dosimetry

- Sophisticated numerical modeling is required to predict distribution of internal fields.
- Today realistic computational models comprising of cubical cells are mostly related to application of Finite Difference Time Domain (FDTD) methods.
- In certain studies, the Finite Element Method (FEM) is considered to be a more accurate method than the FDTD, and a more sophisticated tool for the treatment of irregular or curved shape domains.
- Some studies have demonstrated that the use of Boundary Element Method (BEM), fast multipole techniques and wavelet techniques to reduce the computational cost.
EM and thermal dosimetry: Previous work

Dosimetry: Low and high frequencies

- The induced currents and fields in human organs may give rise to thermal and nonthermal effects.
- When man is exposed to LF fields the thermal effects seem to be negligible, and possible nonthermal effects are related to the cellular level.
- The knowledge of the internal electric field (previously internal current density) is the key to understanding the interaction of the human body with LF fields.
- The key point in HF dosimetry is how much EM energy is absorbed by a biological body and where it is deposited.
- The basic dosimetric quantity for HF fields is the specific absorption rate (SAR).
Dosimetry: Exposure to static fields

**FORMULATION:** Laplace equation

3D electrostatic field distribution between a VDU and the head is governed by the Laplace equation for electric potential $\phi$:

$$\nabla^2 \phi = 0$$

boundary conditions

- $\phi = \phi_s$ on the display
- $\phi = \phi_h$ on the head
- $\nabla \phi \cdot \vec{n} = 0$ on the far field boundaries

3D model of the head located in front of a VDU.
EM and thermal dosimetry: Previous work

**Dosimetry: Exposure to static fields - FEM SOLUTION**

Applying the weighted residual approach to Laplace equation yields:

$$\int_V \nabla^2 \varphi W_j d\Omega = 0$$

Performing some mathematical manipulations gives:

$$\int_V \nabla \varphi \nabla W_j d\Omega = \int_{\Gamma} \frac{\partial \varphi}{\partial n} W_j d\Gamma$$

The Galerkin-Bubnov procedure ($W_j = N_j$) it follows:

$$\int_V \nabla \varphi \nabla N_j d\Omega = \int_{\Gamma} \frac{\partial \varphi}{\partial n} N_j d\Gamma$$

Neumann condition:

$$\frac{\partial \varphi}{\partial n} = 0$$

The unknown potential over an element is expressed by shape functions:

$$\varphi^e = \sum_{i=1}^{4} \alpha_i N_i$$

The shape functions:

$$N_i(x, y, z) = \frac{1}{D} (V_i + a_i x + b_i y + c_i z) \quad i = 1, 2, 3, 4$$

The global matrix system:

$$[a] \{\alpha\} = \{Q\}$$

The electrostatic field:

$$\vec{E} = -\nabla \varphi$$
Dosimetry: Exposure to static fields - Computational examples

Electrostatic field strength [V/cm] on the faces. a) Person 1; b) Person 2

Dosimetry: Exposure to LF fields: Cylindrical body model

\[ E_{inc}^{\text{inc}} = -\frac{1}{4j\pi \omega_0} \int_L \left[ I \frac{\partial^2}{\partial z^2} + k^2 \int g_E(z, z') I(z') dz' + Z_L(z) I(z) \right] dz \]

\[ R = \sqrt{(z - z')^2 + 4a^2 \sin^2(\phi/2)} \]

FEM solution

\[ \sum_{i=1}^\infty [Z]_i = (V)_i \]

and \( j = 1, 2, ..., M \)

\[ [Z]_j = -\frac{1}{4j\pi \omega_0} \left( \int_{\Delta_i} \int_{\Delta_i} \{D\}_j \{D\}^T g_E(z, z') dz' dz + k^2 \int_{\Delta_i} \int_{\Delta_i} \{f\}_j \{f\}^T g_E(z, z') dz' dz + \int Z_L(z) \{f\}_j \{f\}^T dz \right) \]
Dosimetry: Exposure to low frequency fields:
Realistic approach: anatomically based body model

Formulation:
The equation of continuity

\[ \nabla \vec{J} + \frac{\partial \rho}{\partial t} = 0 \]

\[ \vec{J} = -\sigma \nabla \varphi \]

\[ \nabla (\varepsilon \nabla \varphi) = -\rho \]

For the time-harmonic ELF exposures it follows:

\[ \nabla \left[ (\sigma + j \omega \varepsilon) \nabla \varphi \right] = 0 \]

The air-body interface conditions

\[ \vec{n} \times (\nabla \varphi_b - \nabla \varphi_a) = 0 \]

\[ \sigma_b \vec{n} \nabla \varphi_b = -j \omega \rho_s \]

\[ \varepsilon_0 \vec{n} \nabla \varphi_a = \rho_s \]
Numerical method: The Boundary Element Method

The beauty of BEM...

- BEM tends to avoid volume meshes for large-scale problems.
- BEM formulation is based on the fundamental solution of the leading operator for the governing equation thus being competitive with other well-established methods, such as FEM or FDM, in terms of accuracy and efficiency.

The problem consists of finding the solution of the Laplace equation in a non-homogenous media with prescribed boundary conditions:

\[ \nabla \cdot (\sigma \nabla \phi) = 0 \quad \text{on } \Omega \]

\[ \phi = \phi_0 \quad \text{on } \Gamma_1 \]

\[ \frac{\partial \phi}{\partial x_j} n_j = \frac{\partial \phi}{\partial n_j} \quad \text{on } \Gamma_2 \]

The integration domain is considered piecewise homogeneous, so it can be decomposed into an assembly of \( N \) homogeneous subdomains \( \Omega_k (k = 1, m) \).
EM and thermal dosimetry: Previous work

The Boundary Element Method

Green’s theorem yields the integral representation for a subdomain:

\[ c(\xi) \phi(\xi) + \int_{\Gamma_k} \phi \frac{\partial \phi^*}{\partial n} \, d\Gamma = \int_{\Gamma_k} \frac{\partial \phi}{\partial n} \phi^* \, d\Gamma \]

where \( \phi^* \) is the 3D fundamental solution of Laplace equation.

Discretization to \( N_k \) elements leads to an integral relation:

\[ c_i \phi_i + \sum_{j=1}^{N_k} \int_{\Gamma_{k,j}} \phi \frac{\partial \phi^*}{\partial n} \, d\Gamma = \sum_{j=1}^{N_k} \int_{\Gamma_{k,j}} \frac{\partial \phi}{\partial n} \phi^* \, d\Gamma \]

Potential and its normal derivative can be written by means of the interpolation functions \( \psi_a \):

\[ \phi(\xi) = \sum_{a=1}^{6} \psi_a(\xi) \phi_a \]

\[ \frac{\partial \phi(\xi)}{\partial n} = \sum_{a=1}^{6} \psi_a(\xi) \phi_a \]
EM and thermal dosimetry: Previous work

The Boundary Element Method

The system of equations for each subdomain can be written as:

\[ H\phi - G \frac{\partial \phi}{\partial n} = 0 \]

where \( H \) and \( G \) are matrices defined by:

\[
H = h_{ij}^a = \int_{\Gamma_{k,j}} \psi_a \left( \frac{\partial \phi^*}{\partial n} \right)_j d\Gamma
\]

\[
G = g_{ij}^a = \int_{\Gamma_{k,j}} \psi_a \phi^* d\Gamma
\]

The matching between two subdomains can be established through their shared nodes:

\[
\phi^\alpha_j_A = \phi^\alpha_j_B
\]

and

\[
\left( -\tau_A \frac{\partial \phi^\alpha}{\partial n}_j \right)_A = \left( \tau_A \frac{\partial \phi^\alpha}{\partial n}_j \right)_B
\]
EM and thermal dosimetry: Previous work

**Computational Examples:** Exposure to power lines

*The multidomain body of revolution model*

The well-grounded body of 175cm height exposed to the 10kV/m/60Hz power line E-field. The height of the power line is 10m above ground.
The current density values increase at narrow sections such as ankle and neck.

Comparison between the BEM, FEM and experimental results for the current density at various body portions, expressed in [mA/m²]

<table>
<thead>
<tr>
<th>Part of the body</th>
<th>BEM</th>
<th>FEM</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neck</td>
<td>4.52</td>
<td>4.62</td>
<td>4.66</td>
</tr>
<tr>
<td>Pelvis</td>
<td>2.32</td>
<td>2.27</td>
<td>2.25</td>
</tr>
<tr>
<td>Ankle</td>
<td>18.91</td>
<td>19.16</td>
<td>18.66</td>
</tr>
</tbody>
</table>

The calculated results via BEM agree well with FEM and

The main difference is in the area of ankles and neck. The peak values of \( \mathbf{J} \) in those parts maintain the continuity of the axial current throughout the body.

**Exposure scenario**

<table>
<thead>
<tr>
<th>Current density ( \mathbf{J}[\text{mA/m}^2] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICNIRP guidelines for occupational exposure</td>
</tr>
<tr>
<td>ICNIRP guidelines for general public exposure</td>
</tr>
<tr>
<td>( J_{\text{zmax}} ) (cylinder on earth)</td>
</tr>
<tr>
<td>( J_{\text{zmax}} ) (body of revolution model)</td>
</tr>
</tbody>
</table>
EM and thermal dosimetry: Previous work

Computational Examples (cont’d): Exposure

**The realistic models of the human body**

The electric field in the air begins to *sense* the presence of the grounded body at around 5m above ground level.

BEM with domain decomposition and triangular elements (40 000) is used.

A plan view of the integration domain

Electric field in the air near the body

3D mesh: Linear Triangular Elements

Scaled potential lines in air
EM and thermal dosimetry: Previous work

**Computational Examples (cont’d):** Exposure to power lines

Front and side view of equipotential lines in air are presented.

Scaled Equipotential lines in air

An oversimplified cylindrical representation of the body is unable to capture the current density peaks in the regions with narrow cross section.

The presence of peaks in current density values corresponds to the position of the ankle and the neck.

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Figure 1: Axial current density induced in the cylindrical and anatomical body model exposed to ELF external electric field ($E = 10\, \text{kV/m}$, $f = 60\, \text{kHz}$).
EM and thermal dosimetry: Previous work

**Computational Examples (cont’d): Exposure to power**

The mesh and scalar potential for the body model with arms up is presented.

Front and lateral view of equipotential surfaces for the HAU model exposed to a reference incident field $E_z = 0.25 \text{ V/m.}$

The numbers on the left indicate voltage, while the numbers on the right indicate height of the equipotentials taken at 2.5m away from the subject, i.e. when equipotential surface become parallel to the ground.
EM and thermal dosimetry: Previous work

Computational Examples (cont’d): Exposure

Distribution of axial current density along the torso and head in function of the height for the HAU, HAO, HAD and HNA models. The observation line corresponds to the line connecting points A and B.

The bigger cross-sectional area acts as a natural protection to the heart, while the raised arms protect the neck.

Geometry of the arms in HAU, HAO and HAD models

Maximal values of current density are reached by the HAO model, in accordance to the larger area exposed to the normal field.

Absolute value for the axial current density along the arms in function of the HAU, HAO and HAD models.
Axial current density along the legs for the HNA, HAU, HAO and HAD

The maximum values the current density at the height of the ankle are obtained for the HAU and HAO models.

Computational Examples (cont’d): Exposure

Induced current density for the various body models

Comparison between the following body models is presented:

- No arms
- Arms up (60° from horizontal plane)
- Cylinder

Peak values of the \( J_z \) versus \( E \)

<table>
<thead>
<tr>
<th>ICNIRP Safety Standards</th>
<th>( J_z ) [mA/m²]</th>
<th>( E ) [kV/m]</th>
<th>( J_z ) [mA/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Occupational exposure</td>
<td>10</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>19</td>
</tr>
<tr>
<td>General public exposure</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Peak values of the current density in the ankle for some typical values of electric field near ground under power lines are presented in the table.
EM and thermal dosimetry: Previous work

Computational Examples (cont’d): Exposure to power lines

Variations of conductivity in the heterogeneous representation

Tissue conductivities in S/m at 60 Hz

<table>
<thead>
<tr>
<th>Tissue</th>
<th>SETA $\sigma_t/\sigma_m$</th>
<th>SETG $\sigma_t/\sigma_m$</th>
<th>SETD $\sigma_t/\sigma_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Embedding tissue</td>
<td>0.50</td>
<td>1.00</td>
<td>0.22</td>
</tr>
<tr>
<td>Heart</td>
<td>0.11</td>
<td>0.22</td>
<td>0.08</td>
</tr>
<tr>
<td>Brain</td>
<td>0.12</td>
<td>0.24</td>
<td>0.04</td>
</tr>
<tr>
<td>Eye</td>
<td>0.11</td>
<td>0.22</td>
<td>1.0</td>
</tr>
<tr>
<td>Liver</td>
<td>0.13</td>
<td>0.26</td>
<td>0.07</td>
</tr>
<tr>
<td>Intestine</td>
<td>0.16</td>
<td>0.32</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Note on numerical results

- Due to the variation in the conductivity there is a significant variation in the induced $E$-field while the variations of the current density are negligible.
BEM model and exposure results (left). Observation line along the spine of the foetus

**Computational Examples (cont’d): Pregnant woman Exposure...**

**Conductivity Scenarios**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\sigma_f$</th>
<th>Week 8</th>
<th>Week 13</th>
<th>Week 26</th>
<th>Week 38</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.23</td>
<td>0.23</td>
<td>0.23</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{AF}$</td>
<td>1.28</td>
<td>1.27</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_{st}$</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>0.996</td>
<td>0.996</td>
<td>0.574</td>
<td>0.574</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_{AF}$</td>
<td>1.70</td>
<td>1.64</td>
<td>1.64</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_{st}$</td>
<td>0.52</td>
<td>0.52</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>3</td>
<td>0.732</td>
<td>0.732</td>
<td>0.396</td>
<td>0.396</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_{AF}$</td>
<td>1.70</td>
<td>1.64</td>
<td>1.64</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_{st}$</td>
<td>0.17</td>
<td>0.17</td>
<td>0.17</td>
<td>0.17</td>
</tr>
</tbody>
</table>
EM and thermal dosimetry: Previous work

Computational Examples (cont’d): Pregnant woman Exposure...

The uterus, due to its higher conductivity comparing to the maternal tissue, tends to concentrate the field lines.
Dosimetry: HF Exposures

- The key point in HF bioelectromagnetics is how much EM energy is absorbed by a biological body and where it is deposited.

- The basic dosimetric quantity for HF fields is the specific absorption rate (SAR) being defined as the rate of energy $W$ absorbed by or dissipated in a unit mass of the body:

$$\text{SAR} = \frac{dP}{dm} = \frac{d}{dm} \frac{dW}{dt} = C \frac{dT}{dt}$$

where $C$ is the specific heat capacity of tissue, $T$ is the temperature and $t$ is time.

- In tissue, SAR is proportional to the square of the internal electric field strength:

$$\text{SAR} = \frac{dP}{dm} = \frac{dP}{\rho dV} = \frac{\sigma}{\rho} |E|^2$$

where $E$ is the root-mean-square value of the electric field, $\rho$ is the tissue density and $\sigma$ is the tissue conductivity.

- The localized SAR is directly related to the internal field and the main task of dosimetry involves the assessment of the electric field distribution inside the biological body.
EM and thermal dosimetry: Previous work

Dosimetry: HF Exposures

Hybrid Element Method (HEM - BEM/FEM)

Electromagnetic scattering problem – Solution method

Advantages of HEM

- HEM combines the symmetric matrix generated by FEM with the accuracy provided by BIE formulations.
- Efficiently terminates the computational domain.
- Material properties can vary arbitrarily within the computational domain.
- Manipulating the Maxwell equations the time-harmonic EM fields can be expressed as follows:

\[ \nabla \times \left( \frac{1}{\omega \mu} \nabla \times \vec{E} \right) + \left( j \sigma - \omega \varepsilon \right) \vec{E} = 0 \]

\[ \nabla \times \left( \frac{1}{\sigma + j \omega \varepsilon} \nabla \times \vec{H} \right) + j \omega \mu \vec{H} = 0 \]
EM and thermal dosimetry: Previous work

Dosimetry: HF Exposures
Boundary Integral Equations for EM fields

Applying the 2\textsuperscript{nd} Green Theorem yields the integral representations of the $E$ and $H$ fields:

\[
E_z (\vec{r}) = E_z^{\text{inc}} (\vec{r}) + \iint_{S'} \left[ E_z \frac{\partial G (\vec{r}, \vec{r}')}{\partial n} - G (\vec{r}, \vec{r}') \frac{\partial E_z}{\partial n} \right] dS'
\]

\[
H_z (\vec{r}) = H_z^{\text{inc}} (\vec{r}) + \iint_{S'} \left[ H_z \frac{\partial G (\vec{r}, \vec{r}')}{\partial n} - G (\vec{r}, \vec{r}') \frac{\partial H_z}{\partial n} \right] dS'
\]

FEM formulation
Applying the Green theorems to FEM governing equations and featuring the weak formulation of the problem for 2D it follows:

\[
\iint_{\Omega} \left[ \frac{1}{\omega \mu} \nabla E_z \cdot \nabla W_i + (j \sigma - \omega \varepsilon) E_z W_i \right] d\Omega = \int_{\Gamma} \frac{1}{\omega \mu} W_i \frac{\partial E_z}{\partial n} d\Gamma
\]

\[
\iint_{\Omega} \left[ \frac{1}{\sigma + j \omega \varepsilon} \nabla H_z \cdot \nabla W_i + j \omega \mu H_z W_i \right] d\Omega = \int_{\Gamma} \frac{1}{\sigma + j \omega \varepsilon} W_i \frac{\partial H_z}{\partial n} d\Gamma
\]
The principal biological effect of HF exposure is heating of the tissue. Therefore, to quantify hazardous EM field levels thermal response of a human exposed to the HF radiation is also considered.

The bio-transfer equation expresses the energy balance between conductive heat transfer in a volume control of tissue, heat loss due to perfusion effect, metabolism and energy absorption due to radiation.

The stationary bio-heat transfer equation is given by:

\[
\nabla(\lambda \nabla T) + W_b C_{pb} (T_a - T) + Q_m + Q_{EM} = 0
\]
EM and thermal dosimetry: Previous work

**Dosimetry: Thermal response**

The electromagnetic power deposition $Q_{EM}$ is a source term deduced from the electromagnetic modelling, and determined by relation:

$$Q_{EM} = \rho \cdot SAR$$

The inhomogeneous Helmholtz-type equation is given by:

$$\nabla(\lambda \nabla T) - W_b C_{pb} T = -(W_b C_{pb} T_a + Q_m + Q_{EM})$$

The boundary condition for the bio-heat transfer equation, imposed to the interface between skin and air, is given by:

$$q = H(T_s - T_a)$$

where $q$ denotes the heat flux defined as:

$$q = -\lambda \frac{\partial T}{\partial n}$$

while $H$, $T_s$ and $T_a$ denote, respectively, the convection coefficient, the temperature of the skin, and the temperature of the air.
EM and thermal dosimetry: Previous work

Dosimetry: Thermal response

The Bio-heat Transfer Equation: The solution by FEM

Applying the FEM solution the bio-heat transfer equation one obtains the following matrix equation:

\[
[K]\{T\} = \{M\} + \{P\}
\]

The matrix system elements are:

\[
K_{ji} = \int_{\Omega_e} \nabla f_j (\lambda \nabla f_i) d\Omega_e + \int_{\Omega_e} W_b C_{pb} f_j f_i d\Omega_e
\]

\[
M_j = \int_{\Gamma_e} \lambda \frac{\partial T}{\partial n} f_j d\Omega_e
\]

\[
p_{ji} = \int_{\Omega_e} (W_b C_{pb} T_a + Q_m + Q_{EM}) f_j d\Omega_e
\]
EM and thermal dosimetry: Previous work

Dosimetry: HF Exposures
Computational examples: HF exposures

Figure 2  Electric field distribution in the upper left portion of the human head due to TM plane wave at frequency $f = 0.85$ GHz and power density $P_0 = 5.0$ mW cm$^{-2}$.

Figure 3  Logarithmic plot of whole eye averaged SAR for TM plane wave in the frequency range 0.7-4.4 GHz and power density $P_0 = 5$ mW cm$^{-2}$.

Figure 4  Temperature rise in the human eye due to a TM plane wave with frequency $f = 0.85$ GHz and power density $P_0 = 5$ mW cm$^{-2}$.

Figure 5  Logarithmic plot of average and maximum temperature rise in the human eyes due to TM plane wave in the frequency range 0.7-4.4 GHz and power density $P_0 = 5$ mW cm$^{-2}$.
EM and thermal dosimetry: Previous work

Dosimetry: HF Exposures

Computational examples: HF exposures

- the results from hybrid BEM/FEM numerical computation of electric field induced by plane wave with power density of 10 W/m$^2$

*SAR in the eye due to EM wave with power density 10 W/m$^2$ at: (a) f=1 GHz (b) f=2 GHz.*
Temperature rise in the eye due to EM wave with power density 10 W/m² at (a) f=1 GHz (b) f=2 GHz.
EM and thermal dosimetry: Previous work

Dosimetry: Human Exposure to Transient Radiation

Laser Source Modelling

- Laser energy \( H (r, z, t) \), absorbed by the eye tissue at the \( n \)th node with cylindrical coordinates \((r, z)\), is given by a product:

\[
H (r, z, t) = \alpha I (r, z, t)
\]

where:
- \( \alpha \) - is the wavelength dependent absorption coefficient of the specific tissue
- \( I \) - is the irradiance of the \( n \)th node given by:

\[
I (r, z, t) = I_0 \exp \left( - \frac{2r^2}{w^2} - \alpha z \right) \exp \left( - \frac{8t^2}{r^2} \right)
\]

where:
- \( I_0 \) - is the incident value of intensity
- \( w \) - is the beam waist
- \( r \) - is the pulse duration

- Bioheat equation is supplemented with natural boundary condition equations for cornea, sclera and domain inside the eye, respectively:

\[
-k \frac{\partial T}{\partial n} = h_c (T - T_{\text{amb}}) + \sigma \varepsilon (T^4 - T_{\text{amb}}^4) \quad \in \Gamma_1
\]

\[
-k \frac{\partial T}{\partial n} = h_s (T - T_{\text{a}}) \quad \in \Gamma_2
\]

\[
-k \frac{\partial T}{\partial n} = 0 \quad \in \Gamma_3
\]

where:
- \( k \) - specific tissue thermal conductivity
- \( h_c \) - heat transfer coefficient of cornea
- \( h_s \) - heat transfer coefficient of sclera
- \( \varepsilon \) - Stefan–Boltzmann constant
- \( \sigma \) - emissivity of the corneal surface
- \( T_{\text{amb}} \) - temperature of the ambient air anterior to the cornea
- \( T_{\text{a}} \) - arterial blood temperature taken to be 36.7°C

- Second term on the right hand side of Eq. (2) is approximated by \( \sigma \varepsilon T^2 r (T - T_{\text{amb}}) \)

Heat transfer

- The mathematical model is based on the Pennes’ bioheat transfer equation

\[
\rho C_v \frac{\partial T}{\partial t} = \nabla (k \nabla T) + W_b C_p b (T_a - T) + Q_m + H
\]

- Bioheat equation is extended with the new term \( H \) representing heat generated inside the tissue due to laser radiation

Numerical Method

- The equation (1) is discretized in two spatial dimensions and solved using the weak formulation and the Galerkin–Bubnov procedure
- A total number of 21,593 triangular elements and 11,094 nodes were generated using the GID 7.2 mesh generator
- Solving part was done by algorithm written in MATLAB
- The equation is first solved for the steady-state case, i.e. when no external sources are present
- Latter, these results are used as initial conditions in the time domain analysis with included external source, i.e. laser radiation

\[
\begin{align*}
\Delta \left[ nt \right] + \left( W_b C_p b \left( T_a - T \right) + Q_m \right) + \left( h_s C_p b T_{\text{amb}} + h_c C_p b T_{\text{amb}} \right) & \in \Gamma_1 \\
\end{align*}
\]

\[
\begin{align*}
\frac{\partial}{\partial t} \left[ nt \right] + \left( h_s C_p b T_{\text{a}} + h_c C_p b T_{\text{amb}} \right) & \in \Gamma_2 \\
\end{align*}
\]

\[
\begin{align*}
\frac{\partial}{\partial t} \left[ nt \right] + \left( h_s C_p b T_{\text{a}} + h_c C_p b T_{\text{amb}} \right) & \in \Gamma_3
\end{align*}
\]

- Finite element formulation of the equation including the boundary conditions.
EM and thermal dosimetry: Previous work

Dosimetry: Human Exposure to Transient Radiation

**Results** (2090 nm Ho:YAG laser)
- Maximum temperature of 69.424°C, is obtained on cornea-aqueous boundary, as shown on Fig. 6.
- Used in contactless thermal keratoplasty (cornea).

**Results** (193 nm ArF laser, 2nd setup)
- Figure 8: Detail of temperature distribution around anterior part of the eye, due to 15 ns pulse of 193 nm ArF excimer laser.

**Results** (1053 nm Nd:YLF laser)
- Versatile laser, alongside thermal effects, can evoke the plasma-induced ablation and photodisruption.
- Figure 9: Detail of power density distribution around posterior part of the eye.
EM and thermal dosimetry: Ongoing work

- **Scattering** problem
- Brain as a lossy homogeneous dielectric body of arbitrary shape $S$ placed in free space

\[ \varepsilon_{eff} = \varepsilon_0 \varepsilon_r - j \frac{\sigma}{\omega} \]

- Surface integral equation (SIE) formulation
- Equivalence theorem - two equivalent problems
EM and thermal dosimetry: Ongoing work

- **Equivalence theorem** - two equivalent problems (interior and exterior regions) in terms of equivalent currents \( J \) and \( M \) (in order to satisfy b.c. on surface \( S \))

  \[
  \begin{align*}
  \vec{E}^{\text{inc}}, \vec{H}^{\text{inc}} \quad \hat{n} \quad \vec{E}^{\text{sca}}, \vec{H}^{\text{sca}} \\
  \vec{E}_1 = 0 \quad \vec{H}_1 = 0 \quad \vec{E}_2 = 0 \quad \vec{H}_2 = 0 \\
  \varepsilon_1, \mu_1 \quad \varepsilon_2, \mu_2 \\
  S_+ \quad S_- \\
  \vec{J}_1 = \hat{n} \times \vec{H}_1 \quad \vec{M}_1 = -\hat{n} \times \vec{E}_1 \\
  \vec{J}_2 = -\hat{n} \times \vec{H}_2 \quad \vec{M}_2 = \hat{n} \times \vec{E}_2
  \end{align*}
  \]

  **Equivalent problem for exterior**

  \[
  \begin{align*}
  [\vec{E}^{\text{sca}}(\vec{J}, \vec{M})]_{\text{tan}} &= [\vec{E}^{\text{inc}}]_{\text{tan}} \\
  [\vec{H}^{\text{sca}}(\vec{J}, \vec{M})]_{\text{tan}} &= [\vec{H}^{\text{inc}}]_{\text{tan}}
  \end{align*}
  \]

  **Equivalent problem for interior**

  \[
  \begin{align*}
  [\vec{E}^{\text{sca}}(\vec{J}, \vec{M})]_{\text{tan}} &= 0 \\
  [\vec{H}^{\text{sca}}(\vec{J}, \vec{M})]_{\text{tan}} &= 0
  \end{align*}
  \]

- **“Homogenized” space** >> the use of free space Green’s fn.
EM and thermal dosimetry: Ongoing work

- Fields due to these equivalent sources placed in homogeneous space

\[
\tilde{E}_{n}^{\text{sca}}(\vec{J}, \tilde{M}) = -j\omega \vec{A}_{n} - \nabla \varphi_{n} - \frac{1}{\varepsilon_{n}} \nabla \times \vec{F}_{n}
\]

\[
\tilde{H}_{n}^{\text{sca}}(\vec{J}, \tilde{M}) = -j\omega \vec{F}_{n} - \nabla \psi_{n} + \frac{1}{\mu_{n}} \nabla \times \vec{A}_{n}
\]

- \(n=1,2\); index of the region

\[
\vec{A}_{n}(\vec{r}) = \mu_{n} \int_{S} \vec{J}(\vec{r}')G_{n}(\vec{r}, \vec{r}')\, dS'
\]

\[
\varphi_{n}(\vec{r}) = \frac{j}{\omega \varepsilon_{n}} \int_{S} \nabla'_{S} \cdot \vec{J}(\vec{r}')G_{n}(\vec{r}, \vec{r}')\, dS'
\]

\[
\vec{F}_{n}(\vec{r}) = \varepsilon_{n} \int_{S} \vec{M}(\vec{r}')G_{n}(\vec{r}, \vec{r}')\, dS'
\]

\[
\psi_{n}(\vec{r}) = \frac{j}{\omega \mu_{n}} \int_{S} \nabla'_{S} \cdot \vec{M}(\vec{r}')G_{n}(\vec{r}, \vec{r}')\, dS'
\]

- Green’s fn. for homogeneous region \(n\)

\[
G_{n}(\vec{r}, \vec{r}') = \frac{e^{-jk_{n}R}}{4\pi R}; \quad R = |\vec{r} - \vec{r}'| \quad \text{for medium } n
\]

\(k_{n}\) - wave number
EM and thermal dosimetry: Ongoing work

- Scattered field in terms of two equivalent surface currents (fictitious)
- Coupled integro-differential equation set

\[
\begin{align*}
-\hat{E}_{1}^{\text{sca}}(\vec{j}, \vec{M})|_{\tan} &= \left[\hat{E}^{\text{inc}}\right]_{\tan} \\
-\hat{H}_{1}^{\text{sca}}(\vec{j}, \vec{M})|_{\tan} &= \left[\hat{H}^{\text{inc}}\right]_{\tan}
\end{align*}
\]

- Choosing the two b.c. for electric field >> frequency domain integral formulation for dielectric object

**EFIE (electric field integral equation):**

\[
\begin{align*}
-\hat{E}_{n}^{\text{sca}}(\vec{j}, \vec{M})|_{\tan} &= \begin{cases} 
\left[\hat{E}^{\text{inc}}\right]_{\tan}, & n = 1 \\
0, & n = 2
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\hat{E}_{1}^{\text{inc}} &= j\omega \mu_{1} \int_{S} \vec{J}(\vec{r}')G_{1}(\vec{r}, \vec{r}') \, dS' - \\
&\quad - \frac{j}{\omega \varepsilon_{1}} \nabla \int_{S} \nabla' \cdot \vec{J}(\vec{r}')G_{1}(\vec{r}, \vec{r}') \, dS' + \nabla \times \int_{S} \vec{M}(\vec{r}')G_{1}(\vec{r}, \vec{r}') \, dS'
\end{align*}
\]

\[
\begin{align*}
0 &= j\omega \mu_{2} \int_{S} \vec{J}(\vec{r}')G_{2}(\vec{r}, \vec{r}') \, dS' - \\
&\quad - \frac{j}{\omega \varepsilon_{2}} \nabla \int_{S} \nabla' \cdot \vec{J}(\vec{r}')G_{2}(\vec{r}, \vec{r}') \, dS' + \nabla \times \int_{S} \vec{M}(\vec{r}')G_{2}(\vec{r}, \vec{r}') \, dS'
\end{align*}
\]

- \(E^{\text{inc}}\) known, \(J\) and \(M\) unknowns
EM and thermal dosimetry: Ongoing work

- **Numerical solution of EFIE:**
  - expansion of unknown current densities using linear combination of known basis fns. 
    
    \[ \sum_{n=1}^{N} J_n \tilde{f}_n(\mathbf{r}) \]
    
    \[ \sum_{n=1}^{N} M_n \tilde{g}_n(\mathbf{r}) \]

  \[ f_n^\pm(\mathbf{r}) = \begin{cases} 
  \frac{l_n}{2A_n} & , \mathbf{r} \in \mathcal{T}_n^\pm \\
  0 & , \mathbf{r} \notin \mathcal{T}_n^\pm 
  \end{cases} \]

\[ \nabla_S \cdot \tilde{f}_n^\pm(\mathbf{r}) = \begin{cases} 
  \pm \frac{l_n}{A_n} & , \mathbf{r} \in \mathcal{T}_n^\pm \\
  0 & , \mathbf{r} \notin \mathcal{T}_n^\pm 
  \end{cases} \]

\[ \tilde{g}_n(\mathbf{r}) = \hat{n} \times \tilde{f}_n(\mathbf{r}) \]

- \( J_n \) and \( M_n \) - unknown coefficients
- \( N \) - number of triangle pairs (edge elements)
EM and thermal dosimetry: Ongoing work

- Geometrical model of human brain
- Adult average human brain \((l=167\text{mm}, w=140\text{mm}, h=93\text{mm})\)
- Surface discretization by triangles

Development of human brain model a) Sketchup, b) MeshLab, c) Matlab
EM and thermal dosimetry: Ongoing work

- **Numerical solution of EFIE:**
  - multiplying by the set of testing functions $f_m$ ($f_m = RWG$ fn.) and integrating over $S$
  
  $$
  \langle \vec{f}_m, j \omega \vec{A}_i \rangle + \langle \vec{f}_m, \nabla \varphi_i \rangle + \langle \vec{f}_m, \pm \frac{1}{2} \hat{n} \times \vec{M} \rangle + \langle \vec{f}_m, \frac{1}{\varepsilon_i} \nabla \times \vec{E}_n \rangle = \begin{cases} 
  \langle \vec{f}_m, \vec{E}^{inc} \rangle, & i = 1 \\
  0, & i = 2
  \end{cases}
  $$

  - after some mathematical manipulations...
  
  $$
  j \omega \mu_i \sum_{n=1}^{N} J_n \int_{S} \vec{f}_m(\vec{r}) \cdot \int_{S'} \vec{f}_n(\vec{r}') G_i(\vec{r}, \vec{r}') dS' dS + 
  + \frac{j}{\omega \varepsilon_i} \sum_{n=1}^{N} J_n \int_{S} \nabla_S \cdot \vec{f}_m(\vec{r}) \int_{S'} \nabla_{S'} \cdot \vec{f}_n(\vec{r}') G_i(\vec{r}, \vec{r}') dS' dS + 
  + \sum_{n=1}^{N} \sum_{n=1}^{N} M_n \int_{S} \vec{f}_m(\vec{r}) \cdot [\hat{n} \times \vec{g}_n(\vec{r}')] dS + 
  + \sum_{n=1}^{N} M_n \int_{S} \vec{f}_m(\vec{r}) \cdot \int_{S'} \vec{g}_n(\vec{r}') \times \nabla' G_i(\vec{r}, \vec{r}') dS' dS = 
  $$

  - residual term of 3rd integral
  - Cauchy p.v. term of 3rd int.

  - $i=1,2$; index of the region
EM and thermal dosimetry: Ongoing work

- System of linear eqs.: 
  \[ \sum_{n=1}^{N} \left( j \omega \mu_i A_{mn,i} + \frac{j}{\omega \varepsilon_i} B_{mn,i} \right) J_n + \sum_{n=1}^{N} (C_{mn,i} + D_{mn,i}) M_n = \begin{cases} V_m, & i = 1 \\ 0, & i = 2 \end{cases} \]

- Matrix form (2N x 2N):

\[
\begin{bmatrix}
N & N \\
\end{bmatrix}
\begin{bmatrix}
N & \omega \mu_1 A_{mn}^1 + \frac{j}{\omega \varepsilon_1} B_{mn}^1 \\
N & \omega \mu_2 A_{mn}^2 + \frac{j}{\omega \varepsilon_2} B_{mn}^2 \\
C_{mn}^1 + D_{mn}^1 & C_{mn}^2 + D_{mn}^2 \\
\end{bmatrix}
\begin{bmatrix}
J_n \\
M_n \\
\end{bmatrix}
\begin{bmatrix}
V_m^1 \\
V_m^2 \\
\end{bmatrix}
\]

Filling of matrix system:

\[
[Z] \cdot \{I\} = \{V\}
\]

\[
\{I\} = [Z]^{-1} \cdot \{V\}
\]

Solution: coefficients \( J_n \) and \( M_n \)
EM and thermal dosimetry: Ongoing work

- **EM model verification**

  Dielectric sphere of radius 3 cm
  
  \[
  r = 3 \text{ cm}
  \]
  
  \[
  f = 918 \text{ MHz}
  \]
  
  \[
  E_0^{\text{inc}} = 86.83 \text{ V/m}
  \]
  
  \[
  P_0^{\text{inc}} = 1 \text{ mW/cm}^2
  \]
  
  \[
  \varepsilon_r = 35, \sigma = 0.7 \text{ S/m}
  \]
  
  \[
  T = 152, N = 228
  \]

<table>
<thead>
<tr>
<th>Maximum values of equivalent surface currents</th>
<th>SIE</th>
<th>FEKO</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J_{\text{max}} \text{ [mA/m]})</td>
<td>0.454</td>
<td>0.458</td>
</tr>
<tr>
<td>(M_{\text{max}} \text{ [V/m]})</td>
<td>37.7362</td>
<td>38.1</td>
</tr>
</tbody>
</table>
EM and thermal dosimetry: Ongoing work

**EM model verification**

- Dielectric cube of 1m side
  - $a = 1 \text{ m}$, $ka = 1$
  - $f = 47.7456 \text{ MHz}$
  - $E_0^{\text{inc}} = 1 \text{ V/m}$
  - $\varepsilon_r = 54.5$, $\sigma = 1.35 \text{ S/m}$
  - $T = 768$, $N = 1152$

- Maximum values of equivalent surface currents

<table>
<thead>
<tr>
<th></th>
<th>SIE</th>
<th>FEKO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_{\text{max}} \text{ [mA/m]}$</td>
<td>6.3484</td>
<td>6.318</td>
</tr>
<tr>
<td>$M_{\text{max}} \text{ [V/m]}$</td>
<td>0.15063</td>
<td>0.160</td>
</tr>
</tbody>
</table>
EM and thermal dosimetry: Ongoing work

**Human brain exposed to incident plane wave:**
- 900 MHz, VV
- 900 MHz, HH
- 1800 MHz, VV
- 1800 MHz, HH
- \( P = 5 \text{mW/cm}^2 \Rightarrow E = 194.16 \text{V/m} \)

\[ E = \sqrt{2 \eta P}; \quad \eta = 377 \Omega \]

**Calculation of electric field**

\[
\vec{E}_2(\vec{r}) = j \omega \mu_2 \int \int_S \vec{J}(\vec{r}') G_2(\vec{r}, \vec{r}') \, dS' - \frac{j}{\omega \varepsilon_2} \int \int_S \nabla' \cdot \vec{J}(\vec{r}') G_2(\vec{r}, \vec{r}') \, dS' - \int \int_S \vec{M}(\vec{r}') \times \nabla G_2(\vec{r}, \vec{r}') \, dS'
\]

**Calculation of specific absorption rate (SAR)**

\[
\text{SAR} = \frac{\sigma}{2\rho} |\vec{E}|^2
\]
EM and thermal dosimetry: Ongoing work

- **Pennes bioheat equation** [H.H. Pennes, J Appl Physiol, 1948]
  \[
  \rho C \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) + W_b c_b (T_a - T) + Q_m + Q_{EM}
  \]

- **Takes into account:**
  - heat conduction inside tissues
  - heat loss (generation) due to blood flow
  - heat generated due to metabolic processes
  - absorption of incident EM energy

- **Specific absorption rate** (electric field determined from EM model)
  \[
  Q_{EM} = \rho \cdot SAR
  \]
  \[
  SAR = \frac{\sigma}{2\rho} |\vec{E}|^2
  \]
EM and thermal dosimetry: Ongoing work

- **Stationary form:**
  \[ \nabla (\lambda \nabla T) + W_b c_b (T_a - T) + Q_m + Q_{EM} = 0 \]

- **Boundary conditions:**
  \[ -\lambda \nabla T \cdot \hat{n} = h_s (T - T_{amb}) \]
  \( h_s = h_{eff} = 12 \text{ W/m}^2\text{K} \)
  Neumann b.c.
  effective heat transfer coefficient
  [Sustarskii, Yablonskiy, PNAS, 2006]

- **Dirichlet b.c.**
  \( T = 35.4 \text{ C} \)
  Average subdural temperature

- **Numerical solution using FEM** (Finite Elements Method)

Illustracija mreže konačnih elemenata i pripradni rubni uvjeti za površinu mozga
EM and thermal dosimetry: Ongoing work

- Weighted residual approach
- Partially integrating: Gauss’ divergence theorem, etc:

$$\int_{\Omega} (\lambda \nabla T \cdot \nabla W_j + W_b c_b T \cdot W_j) d\Omega = \int_{\Gamma} \lambda \frac{\partial T}{\partial n} \cdot W_j d\Gamma + \int_{\Omega} (W_b c_b T_a + Q_m + Q_{EM}) \cdot W_j d\Omega$$

- Applying the Neumann b.c., integral formulation suitable for FEM:

$$\int_{\Omega} (\lambda \nabla T \cdot \nabla W_j + W_b c_b T \cdot W_j) d\Omega + \int_{\Gamma} h_s T \cdot W_j d\Gamma =$$

$$\int_{\Omega} (W_b c_b T_a + Q_m + Q_{EM}) \cdot W_j d\Omega + \int_{\Gamma} h_s T_a \cdot W_j d\Gamma$$

- Temperature field on one element expanded by known basis fns.

$$T = \sum_{i=1}^{4} \alpha_i N_i$$

$$N_i = \frac{1}{D} (V_i + a_i x + b_i y + c_i z); i = 1, 2, 3, 4$$
EM and thermal dosimetry: Ongoing work

- Testing functions (Galerkin-Bubnov procedure)
- Discretization by the FEM
- Matrix equation for finite element:

\[
[K]^e \{T\}^e = \{M\}^e + \{P\}^e
\]

\[
K_{ij} = \int_{\Omega_e} (\lambda \nabla W_i \cdot \nabla W_j + W_b c_b W_i \cdot W_j) d\Omega_e
\]

\[
P_j = \int_{\Omega_e} (W_b c_b T_a + Q_m + Q_{EM}) W_j d\Omega_e
\]

\[
M_j = \int_{\Gamma_e} \lambda \frac{\partial T}{\partial n} W_j d\Gamma_e
\]

- Assemble the matrix and vectors
- Form the global system

\[
[K] \{T\} = \{M\} + \{P\}
\]

Temperature values in tetrahedra nodes
EM and thermal dosimetry: Ongoing work

- **Electric and magnetic fields on brain surface** *(incident plane wave: $P=5mW/cm^2$)*

<table>
<thead>
<tr>
<th>900 MHz</th>
<th>1800 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td><strong>E_{max}=48.61 V/m, H_{max}=1.571 A/m</strong></td>
<td><strong>E_{max}=89.24 V/m, H_{max}=1.810 A/m</strong></td>
</tr>
<tr>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
<tr>
<td><strong>E_{max}=48.34 V/m, H_{max}=1.179 A/m</strong></td>
<td><strong>E_{max}=69.70 V/m, H_{max}=1.447 A/m</strong></td>
</tr>
</tbody>
</table>
EM and thermal dosimetry: Ongoing work

- Calculation of SAR and temperature increase

<table>
<thead>
<tr>
<th>900 MHz, horizontal polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific absorption rate (SAR) [W/kg]</td>
</tr>
</tbody>
</table>

![Images of brain models showing specific absorption rate (SAR) and temperature increase with different views (rostral, caudal, dorsal, ventral, lateral, medial)]
EM and thermal dosimetry: Ongoing work

- Calculation of SAR and temperature increase

<table>
<thead>
<tr>
<th>900 MHz, vertical polarization</th>
<th>Specific absorption rate (SAR) [W/kg]</th>
<th>Temperature increase (ΔT) [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAR [W/kg]</td>
<td><img src="image1" alt="Rostral view" /></td>
<td><img src="image2" alt="Caudal view" /></td>
</tr>
<tr>
<td><img src="image3" alt="Dorsal view" /></td>
<td></td>
<td><img src="image4" alt="Vertical view" /></td>
</tr>
<tr>
<td><img src="image5" alt="Lateral view" /></td>
<td></td>
<td><img src="image6" alt="Medial view" /></td>
</tr>
</tbody>
</table>

\[ E_{\text{inc}} \rightarrow k \]

COST BM 1309, Split, Croatia,
2 - 3 October 2014
EM and thermal dosimetry: Ongoing work

- Calculation of SAR and temperature increase

1800 MHz, horizontal polarization

<table>
<thead>
<tr>
<th>Specific absorption rate (SAR) [W/kg]</th>
<th>Temperature increase (ΔT) [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>6 x 10^{-3}</td>
</tr>
<tr>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>0.4</td>
<td>4</td>
</tr>
<tr>
<td>0.3</td>
<td>3</td>
</tr>
<tr>
<td>0.2</td>
<td>2</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ \vec{E} \text{ inc} \]
EM and thermal dosimetry: Ongoing work

- Calculation of SAR and temperature increase

<table>
<thead>
<tr>
<th>Specific absorption rate (SAR) [W/kg]</th>
<th>Temperature increase (ΔT) [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1800 MHz, vertical polarization</td>
<td></td>
</tr>
</tbody>
</table>

- [Graph showing SAR and ΔT distribution with views from different angles]
EM and thermal dosimetry: Ongoing work

- **Specific absorption rate in human brain**

<table>
<thead>
<tr>
<th>SAR\textsubscript{max} [W/kg]</th>
<th>900 MHz</th>
<th>1800 MHz</th>
<th>900 MHz</th>
<th>1800 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.856486</td>
<td>0.866016</td>
<td>4.390451</td>
<td>2.678407</td>
<td></td>
</tr>
<tr>
<td>SAR\textsubscript{avg} [W/kg]</td>
<td>0.174457</td>
<td>0.141736</td>
<td>0.158206</td>
<td>0.348032</td>
</tr>
</tbody>
</table>

- **Basic restrictions for professional and general population. According to [ICNIRP, Health Physics, 1998]**

- **Basic restriction** for localized SAR (head & trunk) (10 W/kg) not exceeded for occupational exposure
- For general public population (2 W/kg), exceeded for 1800 MHz
- Whole brain avg. SAR exceeded the basic restriction for professional population (0.4 W/kg) at 1800 MHz, hor. pol.
- Used equivalent power density \( P = 5 \, mW/cm^2 \) represents the worse case than the reference level set for the occupational exposure (\( f/40 \)) for that frequency
Transcranial magnetic stimulation:
Ongoing work

TMS is a method of neuron excitation by means of EM field.

- Non-invasive
- High degree of efficiency
- Not widely used

- Exact biological mechanisms are still not known!!

Typical values of parameters

- Current along the coil: $I_c = 5 \text{ to } 10 \text{ kA}$
- Magnetic induction: $B \approx 2 \text{ T}$
- Induced current density: $J \approx 10 \text{ mA/cm}^2$

Transcranial magnetic stimulation: Ongoing work

- Non-invasive and painless excitation of peripheral and cortical nerves

- Faraday's law:
  \[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}\]

- Applications:
  - Mapping studies
  - Motor pathway testing
  - Tinnitus
  - Schizophrenia
  - Depression, etc.
Human Exposure to EM Fields: Biomedical Applications - TMS

Various TMS coils, arbitrary position

<table>
<thead>
<tr>
<th>Circular</th>
<th>8-coil</th>
<th>Butterfly (10°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2.44 kHz</td>
<td>2.44 kHz</td>
</tr>
<tr>
<td>Radius</td>
<td>4.5 cm</td>
<td>3.5 cm</td>
</tr>
<tr>
<td>Turns</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>Current (max)</td>
<td>2843 A (8kA)</td>
<td>2843 A (8kA)</td>
</tr>
</tbody>
</table>

Tissue parameters (avg. Brain)

\[\varepsilon_r = 46940\]
\[\sigma = 0.08585 \text{ S/m}\]

Development of geometrical brain model

CAD model

Discretised (MeshLab)

MATLAB

Brain model dimensions (average human brain size [Blikov, Glezer 1968])

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>167 mm</td>
<td>140 mm</td>
<td>93 mm</td>
</tr>
</tbody>
</table>

COST BM 1309, Split, Croatia, 2 - 3 October 2014
Transcranial magnetic stimulation: Ongoing work
Formulation: Equivalence Theorem

- Equivalent electric and magnetic currents

\[
\begin{align*}
\vec{J}_1 &= \hat{n} \times \vec{H}_1 \\
\vec{M}_1 &= -\hat{n} \times \vec{E}_1 \\
\vec{J}_2 &= -\hat{n} \times \vec{H}_2 \\
\vec{M}_2 &= \hat{n} \times \vec{E}_2
\end{align*}
\]

- Relationship between scattered and incident fields

\[
\begin{align*}
\left[ -\vec{E}_{1\text{scat}} (\vec{J}, \vec{M}) \right]_{\text{tan}} &= \left[ \vec{E}_{\text{inc}} \right]_{\text{tan}} \\
\left[ -\vec{H}_{1\text{scat}} (\vec{J}, \vec{M}) \right]_{\text{tan}} &= \left[ \vec{H}_{\text{inc}} \right]_{\text{tan}}
\end{align*}
\]
Transcranial magnetic stimulation: Ongoing work

**FORMULATION: Equivalence theorem**

- Scattered fields

\[
\vec{E}_n^{\text{sca}}(\vec{J}, \vec{M}) = -j \omega \vec{A}_n - \nabla \varphi_n - \frac{1}{\varepsilon_n} \nabla \times \vec{F}_n
\]

\[
\vec{H}_n^{\text{sca}}(\vec{J}, \vec{M}) = -j \omega \vec{F}_n - \nabla \psi_n + \frac{1}{\mu_n} \nabla \times \vec{A}_n
\]

- Scalar and vector potential

\[
\vec{A}_n(\vec{r}) = \mu_n \int_S \vec{J}(\vec{r}') G_n(\vec{r}, \vec{r}') \, dS'
\]

\[
\vec{F}_n(\vec{r}) = \varepsilon_n \int_S \vec{M}(\vec{r}') G_n(\vec{r}, \vec{r}') \, dS'
\]

- Green function

\[
G_n(\vec{r}, \vec{r}') = \frac{e^{-jk_n R}}{4\pi R}; \quad R = |\vec{r} - \vec{r}'|
\]

\[
\varphi_n(\vec{r}) = \frac{j}{\omega \varepsilon_n} \int_S \nabla'_S \cdot \vec{J}(\vec{r}') G_n(\vec{r}, \vec{r}') \, dS'
\]

\[
\psi_n(\vec{r}) = \frac{j}{\omega \mu_n} \int_S \nabla'_S \cdot \vec{M}(\vec{r}') G_n(\vec{r}, \vec{r}') \, dS'
\]
Transcranial magnetic stimulation: Ongoing work

**FORMULATION: Equivalence theorem**

- System of electric field integral equations (EFIEs)

\[
\vec{E}_{1}^{\text{inc}} = j\omega\mu_1 \int_S \vec{j}(\vec{r}')G_1(\vec{r}, \vec{r}') \, dS' + \\
\quad + \frac{j}{\omega\varepsilon_1} \int_S \nabla'_{S} \cdot \vec{j}(\vec{r}') \nabla G_1(\vec{r}, \vec{r}') \, dS' + \int_S \vec{M}(\vec{r}') \times \nabla'G_1(\vec{r}, \vec{r}') \, dS'
\]

\[
0 = j\omega\mu_2 \int_S \vec{j}(\vec{r}')G_2(\vec{r}, \vec{r}') \, dS' + \\
\quad + \frac{j}{\omega\varepsilon_2} \int_S \nabla'_{S} \cdot \vec{j}(\vec{r}') \nabla G_2(\vec{r}, \vec{r}') \, dS' + \int_S \vec{M}(\vec{r}') \times \nabla'G_2(\vec{r}, \vec{r}') \, dS'
\]
NUMERICAL SOLUTION: Method of Moments (MoM)

- Approximate solution over the element

\[ \vec{J}(\vec{r}) = \sum_{n=1}^{N} J_n \vec{f}_n(\vec{r}) \]

\[ \vec{M}(\vec{r}) = \sum_{n=1}^{N} M_n \vec{g}_n(\vec{r}) \]

- Base functions

\[ \vec{f}_n(\vec{r}) = \begin{cases} \frac{l_n}{2A_n} \vec{\rho}_n^+, & \vec{r} \in T_n^+ \\ 0, & \vec{r} \notin T_n^\pm \end{cases} \]

\[ \vec{g}_n(\vec{r}) = \hat{n} \times \vec{f}_n(\vec{r}) \]
Transcranial magnetic stimulation: Ongoing work

**NUMERICAL SOLUTION: Method of Moments (MoM)**

\[
j \omega \mu_i \sum_{n=1}^{N} J_n \int_{S} \int_{S'} \vec{f}_m(\vec{r}) \cdot \vec{f}_n(\vec{r}') G_i(\vec{r}, \vec{r}') \, dS' \, dS + \\
+ \frac{j}{\omega \varepsilon_i} \sum_{n=1}^{N} J_n \int_{S} \int_{S'} \vec{f}_m(\vec{r}) \cdot \nabla' S \cdot \vec{f}_n(\vec{r}') \nabla G_i(\vec{r}, \vec{r}') \, dS' \, dS + \\
+ \sum_{n=1}^{N} M_n \int_{S} \int_{S'} \vec{f}_m(\vec{r}) \cdot [\hat{n}' \times \vec{f}_n(\vec{r}')] \times \nabla' G_i(\vec{r}, \vec{r}') \, dS' \, dS = \\
= \left\{ \begin{array}{ll}
\int \vec{f}_m(\vec{r}) \cdot \vec{E}^{\text{inc}} \, dS , & i = 1 \\
0 & , i = 2
\end{array} \right.
\]

\[
\begin{bmatrix}
j \omega \mu_1 A_{mn,1} - \frac{j}{\omega \varepsilon_1} B_{mn,1} \\
\frac{j \omega \mu_2 A_{mn,2} - \frac{j}{\omega \varepsilon_2} B_{mn,2}}
\end{bmatrix}
\begin{bmatrix}
C_{mn,1} + D_{mn,1} \\
C_{mn,2} + D_{mn,2}
\end{bmatrix}
\begin{bmatrix}
J_n \\
M_n
\end{bmatrix}
= 
\begin{bmatrix}
V_m \\
0
\end{bmatrix}
\]
Transcranial magnetic stimulation: Ongoing work

- 1 cm above primary motor cortex
- Comparison of the results

<table>
<thead>
<tr>
<th></th>
<th>Emax [V/m]</th>
<th>Bmax [T]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular</td>
<td>161.153</td>
<td>0.679</td>
</tr>
<tr>
<td>8-coil</td>
<td>321.947</td>
<td>0.672</td>
</tr>
<tr>
<td>Butterfly</td>
<td>328.008</td>
<td>0.8265</td>
</tr>
</tbody>
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<tr>
<td>Circular</td>
<td>86.830</td>
<td>0.750</td>
</tr>
<tr>
<td>8-coil</td>
<td>118.281</td>
<td>0.656</td>
</tr>
<tr>
<td>Butterfly</td>
<td>138.418</td>
<td>0.792</td>
</tr>
</tbody>
</table>
Transcranial magnetic stimulation: Ongoing work

- 1 cm above primary motor cortex
- Comparison of the results

<table>
<thead>
<tr>
<th>Izraz [*]</th>
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<th>Bmax [T]</th>
</tr>
</thead>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Disertacija [***]</th>
<th>Emax [V/m]</th>
<th>Bmax [T]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular</td>
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<td>0.750</td>
</tr>
<tr>
<td>8-coil</td>
<td>118.281</td>
<td>0.656</td>
</tr>
<tr>
<td>Butterfly (10 deg.)</td>
<td>138.418</td>
<td>0.792</td>
</tr>
</tbody>
</table>
Transcranial magnetic stimulation: Ongoing work
Transcranial magnetic stimulation: Ongoing work

**Current density induced in the brain**

<table>
<thead>
<tr>
<th>Maximum values of current densities for three coils</th>
<th>Circular</th>
<th>8-coil</th>
<th>Butterfly (10°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{max}}$ [V/m]</td>
<td>86.8302</td>
<td>1.18.2815</td>
<td>138.4188</td>
</tr>
<tr>
<td>$B_{\text{max}}$ [T]</td>
<td>0.75673</td>
<td>0.85601</td>
<td>0.79243</td>
</tr>
<tr>
<td>$J_{\text{max,ind}}$ [A/m²]</td>
<td>7.483</td>
<td>10.194</td>
<td>11.929</td>
</tr>
</tbody>
</table>

**Direction of the induced currents**

<table>
<thead>
<tr>
<th>TABLE 2. Highest Magnitudes of Current Density on the Surface of the Different Structures at 2.44 kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tissue</td>
</tr>
<tr>
<td>Scalp</td>
</tr>
<tr>
<td>Skull</td>
</tr>
<tr>
<td>Gray matter</td>
</tr>
<tr>
<td>Ventricular system</td>
</tr>
</tbody>
</table>
Transcranial magnetic stimulation: Ongoing work

Induced electric field vector directed parallel to the surface of a brain
Transcranial magnetic stimulation: Ongoing work

An Efficient Model of Transcranial Magnetic Stimulation Based on Surface Integral Equation Formulation

Mario Cvetković, Student Member, IEEE, Dragan Poljak, Senior Member, IEEE and Jens Haneisen, Member, IEEE

| TABLE II |
| COIL PARAMETERS |
| Circular | S-coil | Butterfly |
| Frequency | 2.44 kHz | 2.44 kHz | 2.44 kHz |
| Radius of turn | 4.5 cm | 3.5 cm | 3.5 cm |
| No. of turns | 14 | 15 | 15 |
| Coil current | 2843 A | 2843 A | 2843 A |

Induced $E$ – field at the brain surface

Induced $B$ – field at the brain surface
Concluding remarks

- The use of sophisticated numerical methods is necessary to accurately predict the distribution of TMS induced field in the brain.

- The proposed research activities within the future work will be focused on further improvements of the developed model for the analysis of the human brain exposed to electromagnetic fields.
Concluding remarks

- The specific research activities (RA):

- **RA 1**: development of a more detailed geometrical nonhomogeneous electromagnetic-thermal model of the human brain, that will take into account complex cortical columnar structures, as well as additional tissues such as skin and skull bones.
Concluding remarks

- The specific research activities (RA):

- **RA 2**: the optimization and improvement of the numerical solution method will be undertaken, as well. This will require some comparisons with well established numerical techniques in dosimetry, such as FDTD, FEM and some hybrid techniques (and related trade-off between different numerical methods).
Concluding remarks

- The specific research activities (RA):
  - RA 3: An exhaustive analysis of the most important types of TMS coils by using analytical and numerical procedures will be carried out. Particular emphasis will be given to coil efficiency regarding the deep brain stimulation. It will be also worthwhile to compare the obtained results with the results published in relevant literature.
Concluding remarks

- The specific research activities (RA):

- RA 4: The relationship between TMS-induced electromagnetic fields and the propagation of action potentials along nerve fibers will be investigated. Moreover, an *antenna theory model of the human nerve exposed to an external electromagnetic field excitation* will be developed.
Concluding remarks

- The specific research activities (RA):

- Antenna model of the human nerve exposed to an external electromagnetic field excitation will be based on the corresponding Pocklington integro-differential equations in the frequency domain and related numerical solution via the Galerkin-Bubnov Boundary Element Method (GB-IBEM).

\[
\begin{align*}
    j\omega \frac{\mu}{4\pi} \int_0^L I(x') g(x, x') dx' \\
    - \frac{1}{j4\pi \omega \varepsilon_{eff}} \frac{\partial}{\partial x} \int_0^L \frac{\partial I(x')}{\partial x'} g(x, x') dx' + Z_s(x) I(x) = 0
\end{align*}
\]
Concluding remarks

- The specific research activities (RA):
  - This enhanced nerve model will find the application not only in diagnostics and therapeutic purposes but also in gaining a fundamental knowledge regarding potential adverse effects on human health due to undesired exposure to EM fields.
  - Studies on electrical excitation of nerves, among other aspects will involve; nerve excitation using stimulating electrodes, nerve conduction velocity tests, non-invasive stimulation of nerves via EM fields, external field coupling to nerves due to human exposure to EM radiation sources.
Thank you for your attention.

The practical success of an idea, irrespective of its inherent merit is dependent on the attitude of the contemporaries.

Nikola Tesla

NIKOLA TESLA